Week 1
MATH 34B
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6. Differentiate $y = \dfrac{-5 + \sin x}{x + \cos x}$.

$$y' = \frac{(x+\cos x)\frac{d}{dx}(-5+\sin x) - (-5+\sin x)\frac{d}{dx}(x+\cos x)}{(x+\cos x)^2}$$

$$= \boxed{\frac{(x+\cos x)(\cos x) - (-5+\sin x)(1+(-\sin x))}{(x+\cos x)^2}}.$$

11. Find the equation of the tangent line to the curve $y = \dfrac{-2}{\sin x + \cos x}$ at the point $(0, -2)$.

Use point slope (ie. $y-y_1 = m(x-x_1)$)

To find slope: $y' = \dfrac{(\sin x+\cos x)\frac{d}{dx}(-2) - (-2)\frac{d}{dx}(\sin x+\cos x)}{(\sin x+\cos x)^2}$.

$$= \frac{2(\cos x - \sin x)}{(\sin x+\cos x)^2}.$$

At $x=0$, $y' = \dfrac{2(\cos 0 - \sin 0)}{(\sin 0+\cos 0)^2} = \dfrac{2(1-0)}{(0+1)^2} = 2.$

So, we have given $(0,-2)$, with $y'(0)=2$, the eqn. of tan line is $y-(-2) = 2(x-0)$

$\Rightarrow \boxed{y = 2x-2}$

15. For what values of x in $[0, 2\pi]$ does the graph of $y = \dfrac{\cos x}{2 + \sin x}$ have a horizontal tangent?

y has a horizontal tangent precisely when $y' = 0$.

So, we need to find when $y' = 0$.

We have $y' = \dfrac{(2+\sin x)\frac{d}{dx}\cos x - (\cos x)\frac{d}{dx}(2+\sin x)}{(2+\sin x)^2}$

$= \dfrac{(2+\sin x)(-\sin x) - (\cos x)(\cos x)}{(2+\sin x)^2}$.

Since $\sin x$ always between -1, and 1, denominator never 0!

So, $y' = 0$ when numerator is 0.

$(2+\sin x)(-\sin x) - (\cos x)(\cos x) = 0 \Rightarrow -2\sin x - \sin^2 x - \cos^2 x = 0$.

Since $\cos^2 x + \sin^2 x = 1$, this means $-2\sin x - 1 = 0$. $\Rightarrow \sin x = -\frac{1}{2}$

$\Rightarrow \boxed{x = \frac{7\pi}{6}, 11\pi/6.}$

16. A mass on a spring vibrates horizontally on a smooth level surface (see the figure). Its equation of motion is $x(t) = 1 sint$, where t is in seconds and x in centimeters.

   (a) Find the velocity at time t.

   (b) After finding the velocity of the mass at time $t = 2\pi/3$, in what direction is it moving at that time?

a) velocity = derivative of position

$= x'(t) = \boxed{\cos t.}$

b). We plug $t = 2\pi/3$ into $x'(t) = \cos t$, and see that $x'(2\pi/3) = \cos(2\pi/3) < 0$

This means velocity is negative, which corresponds to the mass retracting (ie. moving left).

36. Differentiate $y = e^{x\cos(x)}$.

$$y' = e^{x\cos x} \frac{d}{dx}(x\cos x)$$

$$= e^{x\cos x}\left(x \frac{d}{dx}\cos x + \cos x \frac{d}{dx} x\right)$$

$$= \boxed{e^{x\cos x}\left(x(-\sin x) + \cos x\right)}$$

38. Differentiate $F(z) = \sin(\frac{z-4}{z+4})$.

$$F'(z) = \cos\left(\frac{z-4}{z+4}\right) \frac{d}{dz}\left(\frac{z-4}{z+4}\right)$$

$$= \cos\left(\frac{z-4}{z+4}\right) \frac{(z+4)\frac{d}{dz}(z-4) - (z-4)\frac{d}{dz}(z+4)}{(z+4)^2}$$

$$= \boxed{\cos\left(\frac{z-4}{z+4}\right) \frac{(z+4) - (z-4)}{(z+4)^2}}$$

45. Differentiate $y = \sqrt{x + \sqrt{x}}$.

$$y' = \frac{1}{2\sqrt{x+\sqrt{x}}} \frac{d}{dx}(x+\sqrt{x})$$

$$= \boxed{\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)}.$$

46. Differentiate $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$.

$$\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{d}{dx}\left(x+\sqrt{x+\sqrt{x}}\right),$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{d}{dx}\sqrt{x+\sqrt{x}}\right).$$

from above...

$$= \boxed{\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)\right)}.$$