

Week 1

MATH 34B

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6. Differentiate $y = \frac{-5 + \sin x}{x + \cos x}$.

$$y' = \frac{(x + \cos x) \frac{d}{dx}(-5 + \sin x) - (-5 + \sin x) \frac{d}{dx}(x + \cos x)}{(x + \cos x)^2}$$

$$= \frac{(x + \cos x)(\cos x) - (-5 + \sin x)(1 + (-\sin x))}{(x + \cos x)^2}$$

11. Find the equation of the tangent line to the curve $y = \frac{-2}{\sin x + \cos x}$ at the point $(0, -2)$.

Use point slope (ie. $y - y_1 = m(x - x_1)$)

$$\text{To find slope: } y' = \frac{(\sin x + \cos x) \frac{d}{dx}(-2) - (-2) \frac{d}{dx}(\sin x + \cos x)}{(\sin x + \cos x)^2}$$

$$= \frac{2(\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$\text{At } x=0, y' = \frac{2(\cos 0 - \sin 0)}{(\sin 0 + \cos 0)^2} = \frac{2(1-0)}{(0+1)^2} = 2$$

So, we have ~~given~~ given $(0, -2)$, with $y'(0) = 2$,
the eqn. of tan line is $y - (-2) = 2(x - 0)$

$$\Rightarrow \boxed{y = 2x - 2}$$

15. For what values of x in $[0, 2\pi]$ does the graph of $y = \frac{\cos x}{2 + \sin x}$ have a horizontal tangent?

y has a horizontal tangent precisely when $y' = 0$.

So, we need to find when $y' = 0$.

We have $y' = \frac{(2 + \sin x) \frac{d}{dx} \cos x - (\cos x) \frac{d}{dx} (2 + \sin x)}{(2 + \sin x)^2}$

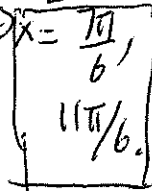
$$= \frac{(2 + \sin x)(-\sin x) - (\cos x)(\cos x)}{(2 + \sin x)^2}$$

Since $\sin x$ always between -1 and 1 , denominator never 0!
 So, $y' = 0$ when numerator is 0.

$$(2 + \sin x)(-\sin x) - (\cos x)(\cos x) = 0 \Rightarrow -2\sin x - \sin^2 x - \cos^2 x = 0.$$

Since $\cos^2 x + \sin^2 x = 1$, this means $-2\sin x - 1 = 0 \Rightarrow \sin x = -\frac{1}{2}$

16. A mass on a spring vibrates horizontally on a smooth level surface (see the figure). Its equation of motion is $x(t) = 1 \sin t$, where t is in seconds and x in centimeters.



- (a) Find the velocity at time t .
 (b) After finding the velocity of the mass at time $t = 2\pi/3$, in what direction is it moving at that time?

a) velocity = derivative of position
 $= x'(t) = \cos t$

b). We plug $t = 2\pi/3$ into $x'(t) = \cos t$, and see that $x'(2\pi/3) = \cos(2\pi/3) < 0$

This means velocity is negative, ~~over~~ which corresponds to the mass retracting (ie. moving left).

36. Differentiate $y = e^{x \cos(x)}$.

$$\begin{aligned}y' &= e^{x \cos x} \frac{d}{dx} (x \cos x) \\&= e^{x \cos x} \left(x \frac{d}{dx} \cos x + \cos x \frac{d}{dx} x \right) \\&= \boxed{e^{x \cos x} (x(-\sin x) + \cos x)}\end{aligned}$$

38. Differentiate $F(z) = \sin\left(\frac{z-4}{z+4}\right)$.

$$\begin{aligned}F'(z) &= \cos\left(\frac{z-4}{z+4}\right) \frac{d}{dz} \left(\frac{z-4}{z+4}\right) \\&= \cos\left(\frac{z-4}{z+4}\right) \frac{(z+4) \frac{d}{dz} (z-4) - (z-4) \frac{d}{dz} (z+4)}{(z+4)^2} \\&= \boxed{\cos\left(\frac{z-4}{z+4}\right) \frac{(z+4) - (z-4)}{(z+4)^2}}\end{aligned}$$

45. Differentiate $y = \sqrt{x + \sqrt{x}}$.

$$y' = \frac{1}{2\sqrt{x+\sqrt{x}}} \frac{d}{dx}(x + \sqrt{x})$$
$$= \boxed{\frac{1}{2\sqrt{x+\sqrt{x}}} \left(1 + \frac{1}{2\sqrt{x}}\right)}$$

46. Differentiate $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$.

$$\frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \cdot \frac{d}{dx}(x + \sqrt{x + \sqrt{x}})$$
$$= \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \left(1 + \frac{d}{dx} \sqrt{x + \sqrt{x}}\right)$$

from above...

$$= \boxed{\frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \left(1 + \frac{1}{2\sqrt{x + \sqrt{x}}} \left(1 + \frac{1}{2\sqrt{x}}\right)\right)}$$